ON WARING'S PROBLEM FOR CUBES.

By

H. DAVENPORT

of MANCHESTER.

Introduction.

The object of this paper is to give a proof of the following

Theorem. *Almost all positive integers are representable as the sum of four positive integral cubes. More precisely, if $E(N)$ denotes the number of positive integers less than N that are not so representable, then*

$$E(N) = O\left(N^{1-\frac{1}{30}+\varepsilon}\right)$$

as $N \to \infty$, for any $\varepsilon > 0$.

It was proved by Hardy and Littlewood[1] that almost all positive integers are representable as the sum of five positive integral cubes. The new weapon which is necessary in order to improve on this is provided by Lemma 1 below.

It is not true that almost all positive integers are sums of three positive integral cubes. This can be seen in two different ways. Firstly, since any cube is congruent to 0, 1, or -1 (mod 9), the sum of three cubes cannot be congruent to 4 or 5 (mod 9). Secondly, the number of integral solutions of

$$x^3 + y^3 + z^3 \leqq N,$$
$$x \geqq y \geqq z > 0,$$

is easily found to be asymptotically

$$\frac{1}{6}\left(\Gamma\left(\frac{4}{3}\right)\right)^3 N$$

---

[1] Partitio Numerorum VI, *Math. Zeitschrift*, 23 (1925), 1—37.