ON CHARACTER SUMS IN FINITE FIELDS.

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1. Introduction.

Let $q = p^e$ be a power of a prime p, and let $[q]$ denote the finite field (or "Galois field") of q elements. Let $f_1(x), \ldots, f_r(x)$ be polynomials over $[q]$, and let $\chi_1, \ldots, \chi_r$ be multiplicative characters of $[q]$ with the convention $\chi(0) = 0$. A character sum is an expression of the form

$$S(f, \chi) = \sum_{x \text{ in } [q]} \chi_1(f_1(x)) \ \ldots \ \chi_r(f_r(x)). \tag{1}$$

We shall make the (trivial) simplification of supposing that $\chi_1, \ldots, \chi_r$ are non-principal characters, and that $f_1(x), \ldots, f_r(x)$ are different normalised[1] polynomials, each irreducible over $[q]$. Let $k_1, \ldots, k_r$ denote the degrees of these polynomials, and let $K = k_1 + \cdots + k_r$.

In connection with any such character sum we define a function $L(f, \chi; s)$ of the complex variable $s = \sigma + it$ which is in fact a polynomial in q^{-s} of degree $K - 1$. These L-functions are essentially the same as those obtained by Hasse[2] as factors of the congruence zeta-function of an algebraic function-field generated by an equation of the form $y^n = f(x)$. The object of this paper is to give a more direct and elementary account of these L-functions.

The definition is as follows. Let (f, g) denote the resultant of two normalised polynomials $f(x)$, $g(x)$ over $[q]$.[3] Let

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[1] A normalised polynomial is one in which the coefficient of the highest power of x is 1.

[2] Journal für Math. (Crelle), 172 (1934), 37—54.

[3] $(f, g) = \prod_{\Phi} f(\Phi)$, where Φ runs through the roots of $g(x) = 0$.