NECESSARY CONDITIONS IN THE CALCULUS OF VARIATIONS.

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§ 1. **Introduction.** The object of this note is to introduce new variational methods, which belong to the order of ideas of MINKOWSKI's theorem on the existence of a flat support for a convex figure. We here apply these methods to the inhomogeneous form of the »simplest problem» of the Calculus of Variations, and we establish, by means of them, a necessary condition, of a very general form, for an attained minimum. In the proof, an important part is also played by theorems of measurability which belong to the theory of Analytic and Projective Sets.

Our necessary condition generalizes at the same time the necessary condition of WEIERSTRASS, the equations to an extremal of EULER and DU BOIS REYMOND, and the equation $\Omega(x, y) = 0$ to certain limiting solutions of CARATHEODORY.

Our methods enable us to dispense with many classical restrictions on the integrand $f(x, y, y')$. The important restrictions of TONELLI on the existence and order of magnitude of the partial derivative f_y are replaced by weaker restrictions on the behaviour of the corresponding partial finite-difference ratio. Apart from these weakened restrictions which concern only the dependence of f on the variable y, our integrand may be any function measurable (B).

We use integration consistently in the general DENJOY sense. This corresponds to an enlargement of the class of admissible curves. We consider also a still further enlargement obtained by admitting what we term *generalized curves.*

In variational problems such as we treat here, in which no restriction is imposed on the order of magnitude of f_y, these enlargements of the class of admissible curves do not necessarily lead to corresponding generalizations of the classical problems. It is easily seen, however, that our methods apply also — and are indeed slightly simplified — when classical restrictions are imposed on the