LINEAR DIFFERENTIAL EQUATIONS WITH ALMOST PERIODIC COEFFICIENTS.

BY

ROBERT H. CAMERON.[1]
in CAMBRIDGE, MASS.

§ 1. Introduction.

1.1. We shall deal with the system of differential equations

$$\frac{d\,\xi_1(t)}{dt} = \alpha_{11}(t)\,\xi_1(t) + \cdots + \alpha_{1n}(t)\,\xi_n(t) + \beta_1(t)$$

$$(1.11) \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{d\,\xi_n(t)}{dt} = \alpha_{n1}(t)\,\xi_1(t) + \cdots + \alpha_{nn}(t)\,\xi_n(t) + \beta_n(t);$$

in which the functions $a_{\nu\nu}(t)$ and β_μ are real or complex a. p.[2] functions of the real variable t, and the $\beta_\mu(t)$ may or may not be identically zero. We shall seek to determine conditions under which the solutions of (1.11) are of a rather general type involving a. p. functions. Before characterizing this type of solution more explicitly, we shall introduce a shorter vector terminology.

1.2. Troughout this paper we shall use the letters x, y, z, and b to denote n-dimensional vectors (or matrices of n rows and one column) having the components $\xi_1, \ldots, \xi_n$; $\eta_1, \ldots, \eta_n$; $\zeta_1, \ldots, \zeta_n$; and $\beta_1, \ldots, \beta_n$. The vector $\frac{d}{dt}\xi_1(t), \ldots, \frac{d}{dt}\xi_n(t)$ will be denoted by $D[x]$; the n-by-n matrix whose elements are $\alpha_{\mu\nu}$ will be denoted by A, and the matrix product of A and x will be denoted by $A \,.\, x$. Hence in this terminology (1.11) becomes

$$(1.21) \qquad D[x(t)] = A(t) \cdot x(t) + b(t).$$

We shall also define a norm for vectors, namely $\|x\| = |x_1| + \cdots + |x_n|$.

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[2] a. p. = almost periodic (in Bohr's sense).