A MAXIMAL THEOREM WITH FUNCTION-THEORETIC APPLICATIONS.

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I.

Introduction.

1. The kernel of this paper is 'elementary', but it originated in attempts, ultimately successful, to solve a problem in the theory of functions. We begin by stating this problem in its apparently most simple form.

Suppose that $\lambda > 0$, that

$$f(z) = f(re^{i\theta})$$

is an analytic function regular for $r \leqq 1$, and that

$$F(\theta) = \operatorname*{Max}_{0 \leqq r \leqq 1} |f(re^{i\theta})|$$

is the maximum of $|f|$ on the radius θ. Is it true that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} F^{\lambda}(\theta)\,d\theta \leqq A(\lambda)\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(e^{i\theta})|^{\lambda}\,d\theta,$$

where $A(\lambda)$ is a function of λ only? The problem is very interesting in itself, and the theorem suggested may be expected, if it is true, to have important applications to the theory of functions.

The answer to the question is affirmative, and is contained in Theorems 17 and 24—27 below (where the problem is considered in various more general