REGULAR AND SEMI-REGULAR POSITIVE TERNARY QUADRATIC FORMS.

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1. *Introduction.* For any ternary quadratic form $f(x, y, z)$ with integral coefficients there are usually congruences $f \equiv h \pmod{m}$ which are not solvable, whence no number $mn + h$ is represented by f, where n is an integer. For instance, $f = x^2 + y^2 + z^2 \equiv 3 \pmod{4}$ implies that x, y and z are odd, whence $f \equiv 3 \pmod{8}$. It follows that f represents no number $8n + 7$ where n is an integer. Similarly f may be shown to represent no number $4^k(8n + 7)$. In this case, these are the only numbers congruentially excluded. For any form the numbers so excluded consist of certain arithmetic progressions of the forms $2^r(8n + a)$, $p^s(pn + b)$, where r and s range over some or all non-negative integers, a is odd, p is an odd prime factor of the determinant of f, and b ranges over the quadratic residues or non-residues of p or both. H. J. S. Smith's definition of *genus*[1] in terms of the characters $(f \mid p)$ etc., of the form and its reciprocal, is equivalent[2] to the following: two forms of the same determinant are in the same genus if the progressions associated, as above, with the forms are the same. Two forms are of the same genus, as proved by H. J. S. Smith, if and only if one can be carried into the other by a linear transformation of determinant 1 and whose coefficients are rational numbers whose denominators are prime to twice the determinant of the forms. It is therefore natural in this article that the solution of problems in genera of several classes[3] is found by use of such

[1] H. J. S. SMITH, *Collected Papers*, vol. 1, pp. 455—509; *Philosophical Transactions*, vol. 157, pp. 255—298.

[2] B. W. JONES, *Trans. Amer. Math. Soc.*, vol. 33 (1931), pp. 92—110; also ARNOLD ROSS *Proc. Nat. Acad. Sc.*, vol. 18 (1932), pp. 600—608.

[3] Two forms are of the same *class* if one may be taken into the other by a linear trans formation with *integral* coefficients and of determinant 1; i. e. by a unimodular transformation.