ON THE UNSYMMETRIC TOP.*

By

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The problem of the motion of a heavy rigid body about a fixed point is an old problem, — one of which much has been written but of which little is known. Euler[1] first stated the equations of motion in the final definitive and elegant form in use today. They are

$$(1) \qquad I_1 \dot{\omega}_1 + (I_3 - I_2)\,\omega_2\,\omega_3 = H_1$$

$$(2) \qquad I_2 \dot{\omega}_2 + (I_1 - I_3)\,\omega_1\,\omega_3 = H_2$$

$$(3) \qquad I_3 \dot{\omega}_3 + (I_2 - I_1)\,\omega_1\,\omega_2 = H_3.$$

The angular velocities ω_1, ω_2 and ω_3 are connected with Euler's angles Θ, Φ and Ψ by the equations:

$$(4) \qquad \dot{\Theta} = \omega_1 \cos \Phi - \omega_2 \sin \Phi$$

$$(5) \qquad \dot{\Phi} = -\omega_1 \sin \Phi \cot \Theta - \omega_2 \cos \Phi \cot \Theta + \omega_3$$

$$(6) \qquad \dot{\Psi} = \omega_1 \sin \Phi \csc \Theta + \omega_2 \cos \Phi \csc \Theta$$

or by the equations:

$$(7) \qquad \omega_1 = \dot{\Theta} \cos \Phi + \dot{\Psi} \sin \Theta \sin \Phi$$

$$(8) \qquad \omega_2 = -\dot{\Theta} \sin \Phi + \dot{\Psi} \sin \Theta \cos \Phi$$

$$(9) \qquad \omega_3 = \dot{\Phi} + \dot{\Psi} \cos \Theta.$$

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[1] Euler: Mémoires de L'Académie de Berlin, 1758.