THE COMBINATORIAL TOPOLOGY OF ANALYTIC FUNCTIONS ON THE BOUNDARY OF A DISK

BY

CHARLES J. TITUS

University of Michigan, U.S.A.

PART I. THE GEOMETRIC PROBLEM

1. Preliminaries

A mapping $\zeta : M^1 \to E^2$, with M^1 an oriented 1-manifold (differentiable 1-manifold), is called a *representation* (*regular representation* if it possesses a continuous non-vanishing tangent ζ'); such a mapping will be described by complex valued function $\zeta(t) = \xi(t) + i\eta(t)$. An image point ζ_0 of a regular representation is a *simple crossing point* if there exist exactly two distinct points t_0' and t_0'' such that

$$\zeta(t_0') = \zeta(t_0'') = \zeta_0$$

and the tangents $\zeta'(t_0')$ and $\zeta'(t_0'')$ are linearly independent. A regular representation is *normal* (Whitney [12], p. 281) if it has a finite number of simple crossing points and for every other image point ζ has but one pre-image point t. A pair of representations (regular representations) ζ^1 and ζ^2 are *equivalent* if there exists a sense-preserving homeomorphism φ of M^1 onto M^1 such that $\zeta^2 = \zeta^1 \circ \varphi$ (and if $\varphi'(t)$ is continuous with $\varphi'(t) \neq 0$). With this equivalence relation one may define a *regular* (*normal*) *curve* as an oriented curve with a regular (normal) representation.

A mapping $F: M^2 \to E^2$, with M^2 a 2-manifold, is *open* if for every open set U in M^2 the set $F(U)$ is open in E^2; F is *light* if the pre-image of every point is totally disconnected; F is *interior* if F is light and open.

THEOREM (Stoilow [8], p. 121). *For every interior mapping F of a manifold M^2 into the complex plane there exists a homeomorphism H of M^2 onto a Riemann Surface R and an analytic function W of R into the complex plane such that $F = W \circ H$.*