CONNECTIONS AND CONFORMAL MAPPING

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Introduction

1. In the theory of conformal mapping and of Riemann surfaces, the concepts of invariance and covariance under change of variable play an important role. They allow the study of functions and differentials on abstractly given domains and have been extensively utilized from the early days of the theory. In this development the complete analogy to the tensor calculus, in general differential geometry of surfaces, has been helpful and has motivated and guided the investigations. A differential in the theory of Riemann surfaces is the analogue of a tensor in differential geometry in so far as both entities are transformed by a linear homogeneous operation under change of the coordinate system. However, it is well known that differential geometers were soon led to introduce entities with more complicated transformation laws than those of tensors. In particular, the Christoffel symbols and connections of a surface became an important tool in the study of the geodesics and the curvature of a surface. A connection is an entity which transforms under a linear but non-homogeneous law if the coordinates are changed. It is natural to inquire whether the analogous concept of a connection should be applied likewise in the theory of conformal mapping and Riemann surfaces. The present paper is devoted to an exposition of the role of connections in various applications of this kind. Before entering into a systematic development of the theory of connections, we wish to give in this introduction a brief preview of our results. This will enable the reader to judge at one glance the usefulness and significance of the concept of connection in a systematic study of conformal transformations.

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(1) This work was supported in part by the Office of Naval Research under Task Nr 041-086 and by the National Science Foundation, Grant 8199, at Stanford University. Reproduction in whole or in part is permitted for any purpose of the United States Government.