ON THE COHOMOLOGY OF TWO-STAGE POSTNIKOV SYSTEMS

BY

LEIF KRISTENSEN

University of Chicago, U.S.A. and Aarhus University, Denmark

1. Introduction

In the recent development of algebraic topology, cohomology operations have proved to be of vital importance. Several examples of such operations (primary and higher orders) have been constructed by Adem, Massey, Pontrjagin, Steenrod, Thomas, and others. A cohomology operation (primary) relative to dimensions q, $q+i$ and coefficient groups G_1, G_2 is a natural transformation (see Eilenberg–MacLane [5]) between the cohomology functors $H^q(\ ,G_1)$ and $H^{q+i}(\ ,G_2)$ defined on the category of topological spaces:

$$\theta: H^q(\ ,G_1)\to H^{q+i}(\ ,G_2).$$

Primary cohomology operations are closely connected with the cohomology of Eilenberg–MacLane spaces (see Serre [11]). The secondary operations are in a similar way connected with the cohomology of spaces with two non-vanishing homotopy groups and of spaces closely connected with these. This has been shown in works by Adams [1] and Peterson–Stein [10].

In this paper we shall compute the cohomology of certain spaces $P_{n,h}$ (see below) with two non-vanishing homotopy groups. This computation is carried out by means of a spectral sequence argument.

The spectral sequence argument giving the cohomology of $K(\pi, n)$'s (see Serre [11]) relies heavily on the fact the transgression commutes with Steenrod operations. In the computation of $H^*(P_{n,h})$ this however does not suffice. We need to have some information about the differentials of $\mathrm{Sq}^1\alpha$, where $\alpha\in H^*(F)$, F the fibre in a fibration $E\to B$, even if α is not transgressive, provided the differentials on α are known. Sections 3–8 in this paper are devoted to the study of this and of related problems.