THE WIENER–HOPF EQUATION IN AN ALGEBRA OF BEURLING

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This paper treats the Wiener–Hopf equation in the framework of a Banach convolution algebra A^2 of functions on the real line that has been constructed and investigated by Beurling, by whose permission we here reproduce part of that theory. The author also acknowledges with much pleasure his great indebtedness to Professor Beurling for his guidance in this research.

We use few specialized notations and conventions. Our function spaces are all spaces of complex valued functions on the real line, hence we denote by L^1 (L^2) the space of all functions that are integrable (square integrable) over the whole real line. When an integral sign carries no limits, the integral is understood to be taken over the whole real line. The elements of the two principal function spaces A^2 and B^2 (the Banach space dual of A^2) are also distinguished by letting Latin minuscules denote elements of A^2 and Greek minuscules elements of B^2. The norm sign $\|f\|$, $\|\varphi\|$ is only used for the norms of A^2 and B^2. By the above convention no ambiguity may arise therefrom. All infinite sequences are indexed by the natural numbers and sum and product signs with no limits mean sum and product respectively over the whole set of natural numbers (we use only absolutely convergent sums and products).

Since the original paper of Wiener and Hopf [6] many authors have worked to remove its growth restrictions on the kernel of the equation. The case when the Fourier transform of the kernel has no real zeros is treated in a recent big expository paper by Krein [4]. The case when the transform of the kernel is real and has real zeros has been treated by Widom [5] and is also the object of the present paper. The essential feature of this study is that it yields, for kernels $f \in A^2$ that satisfy certain additional restrictions on its Fourier transform $\hat{f}$, all solutions in B^2 of the Wiener–Hopf equation. A similar study was made earlier by Beurling [2], using still