SOME PROBLEMS OF DIOPHANTINE APPROXIMATION: AN ADDITIONAL NOTE ON THE TRIGONOMETRICAL SERIES ASSOCIATED WITH THE ELLIPTIC THETA-FUNCTIONS.

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1. In this note we give an alternative and more instructive proof of the fundamental theorem on which our earlier researches in this field[1] were based. The theorem may be stated as follows.

Theorem A. *Suppose that*

(1. 1) $$0<x<1, \quad 0\leqq\theta\leqq 1, \quad \omega>1$$

and

(1. 2) $$s(\omega)=s(\omega, x, \theta)=\sum_{0\leqq n^2\leqq\omega} e^{-n^2\pi i x}\cos 2n\pi\theta.$$

Then

(1. 3) $$s(\omega, x, \theta)-\frac{e^{-\frac{1}{4}\pi i}}{\sqrt{x}}e^{\frac{\pi i\theta^2}{x}}s\left(x^2\omega, -\frac{1}{x}, \frac{\theta}{x}\right)=O\left(\frac{1}{\sqrt{x}}\right)$$

uniformly in ω and θ.[2]

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[1] G. H. Hardy and J. E. Littlewood, 'Some problems of Diophantine Approximation', *Acta mathematica,* 37 (1914), 193—238, and *Proc. Cambridge Phil. Soc.,* 21 (1923), 1—5.

[2] That is to say, the absolute value of the left hand side is less than $Ax^{-\frac{1}{2}}$, where A is an absolute constant.