CONTRIBUTIONS TO THE THEORY OF THE RIEMANN ZETA-FUNCTION AND THE THEORY OF THE DISTRIBUTION OF PRIMES

BY

G. H. HARDY and J. E. LITTLEWOOD,

TRINITY COLLEGE, CAMBRIDGE.[1]

I.

Introduction and summary.

1. 1.[2] We have united in this paper a series of contributions towards the solution of various outstanding questions in the Analytic Theory of Numbers.

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[1] Some of the results of which this memoir contains the first full account have already been stated shortly and incompletely in the following notes and abstracts.

G. H. HARDY: (1) 'On the zeros of RIEMANN's Zeta-function', *Proc. London Math. Soc.* (records of proceedings at meetings), ser. 2, vol. 13, 12 March 1914, p. xxix; (2) 'Sur les zéros de la fonction $\zeta(s)$ de RIEMANN', *Comptes Rendus*, 6 April 1914.

J. E. LITTLEWOOD: 'Sur la distribution des nombres premiers', *Comptes Rendus*, 22 June 1914.

G. H. HARDY and J. E. LITTLEWOOD: (1) 'New proofs of the prime-number theorem and similar theorems', *Quarterly Journal*, vol. 46, 1915, pp. 215—219; (2) 'On the zeros of the RIEMANN Zeta-function' and (3) 'On an assertion of TSCHEBYSCHEF', *Proc. London Math. Soc.* (records etc.), ser. 2, vol. 14, 1915, p. xiv.

[2] The sections, paragraphs, and formulae contained in this memoir are numbered according to the decimal system of PEANO, the aggregate of numbers employed forming a selection of the rational numbers arranged in order of magnitude. Thus every number occurring in the first section begins with 1; the first paragraph is 1. 1 and the first formula of the first paragraph 1. 11. The second would naturally be 1. 12; but here four formulae occur which are parallel for the purposes of our argument, and so these are numbered 1. 121, 1. 122, 1. 123 and 1. 124.

In a long and complicated memoir such as this, PEANO's system has many advantages. It enables the author, in the process of revision of his work, to delete or insert formulae without serious interference with the numbering of the remainder; and it enables the reader to discover any formula referred to with the minimum of trouble.