p-RINGS AND THEIR BOOLEAN-VECTOR REPRESENTATION.

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1. **Introduction.** In a series of earlier papers[1] [1], ..., [7] both the (simple) duality theory of rings, and later the general K-ality theory (not only of rings, but of arbitrary operational disciplines), corresponding to a preassigned group K of admissible "coordinate transformations" in the ring (or discipline) were introduced and studied. Among the interesting concepts which were shown to evolve from this general theory is the notion "ring-logic",—or "ring-algebra" (mod K). In this connection the class of "p-rings" was shown to possess an enveloping "p-ality theory" which generalizes the familiar duality of Boolean rings (and algebras),—which latter are simply 2-rings ($p = 2$). Furthermore for the special cases $p = 2$ and $p = 3$, it was explicitly shown in [1] and [4] that such p-rings are ring-logics (mod N), where N is the "natural group" (see § 2),—with the status of general p-rings, i. e., $p > 3$, left undecided.

Here, for given p (= prime), a p-ring,—as first introduced by Mc Coy and Montgomery [8], is a commutative ring with unit[2] in which for all elements a,

$$a^p = a \tag{1.1}$$

$$pa = 0\,. \tag{1.2}$$

The concept p-ring is an evident generalization of that of Boolean ring ($p = 2$). (In this Boolean case, $p = 2$, the condition (1.2) is a familiar consequence of (1.1); however for $p > 2$ (1.2) is independent).

The following well known result of Stone [9]:

(1°) *Each Boolean ring is isomorphically representable as a ring of classes, or,*

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[1] Square brackets refer to the appended bibliography.

[2] For Mc Coy and Montgomery, [8], the notion "p-ring" does not demand the existence of a unit.