THE APPROXIMATION TO ALGEBRAIC NUMBERS BY RATIONALS.

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I. The object of this paper is the proof of the following

Theorem. *If θ is any algebraic number of degree $n \geq 2$, and if there are an infinite number of rational fractions p/q such that*

(1) $$|\theta - (p/q)| < q^{-\mu},$$

then

(2) $$\mu \leq \sqrt{(2n)}.$$

There is a famous theorem due to Siegel (**1**) which states that under the same hypotheses μ satisfies the weaker inequality

(3) $$\mu \leq \min_{1 \leq s \leq n-1} \left(s + \frac{n}{s+1} \right) < 2\sqrt{n}.$$

It is easy to see that (2) is stronger than (3) for all $n > 2$, although the improvement is not great for small values of n.

The history of previous attempts to obtain a stronger result than Siegel's is a curious chapter of accidents, and will be briefly summarised here before proceeding to the proof of the theorem stated above. In the first place it is probable, and was conjectured by Siegel, that the correct conclusion to be drawn from the hypotheses of the theorem is

(4) $$\mu \leq 2,$$

irrespective of the degree n of θ. In this direction, Siegel (**2**) proved that if (1) is satisfied for an infinite sequence of fractions p_i/q_i with