

# SOME PROBLEMS OF 'PARTITIO NUMERORUM'; III: ON THE EXPRESSION OF A NUMBER AS A SUM OF PRIMES.

By

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## I. Introduction.

1. 1. It was asserted by GOLDBACH, in a letter to EULER dated 7 June, 1742, that *every even number  $2m$  is the sum of two odd primes*, and this proposition has generally been described as 'Goldbach's Theorem'. There is no reasonable doubt that the theorem is correct, and that the number of representations is large when  $m$  is large; but all attempts to obtain a proof have been completely unsuccessful. Indeed it has never been shown that every number (or every large number, any number, that is to say, from a certain point onwards) is the sum of 10 primes, or of 1 000 000; and the problem was quite recently classified as among those 'beim gegenwärtigen Stande der Wissenschaft unangreifbar'.<sup>1</sup>

In this memoir we attack the problem with the aid of our new transcendental method in 'additiver Zahlentheorie'.<sup>2</sup> We do not solve it: we do not

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<sup>1</sup> E. LANDAU, 'Gelöste und ungelöste Probleme aus der Theorie der Primzahlverteilung und der Riemannschen Zetafunktion', *Proceedings of the fifth International Congress of Mathematicians*, Cambridge, 1912, vol. 1, pp. 93—108 (p. 105). This address was reprinted in the *Jahresbericht der Deutschen Math.-Vereinigung*, vol. 21 (1912), pp. 208—228.

<sup>2</sup> We give here a complete list of memoirs concerned with the various applications of this method.

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1. 'Asymptotic formulae in combinatory analysis', *Comptes rendus du quatrième Congrès des mathématiciens Scandinaves à Stockholm*, 1916, pp. 45—53.

2. 'On the expression of a number as the sum of any number of squares, and in particular of five or seven', *Proceedings of the National Academy of Sciences*, vol. 4 (1918), pp. 189—193.