ON THE OVERCONVERGENCE OF CERTAIN SEQUENCES OF RATIONAL FUNCTIONS OF BEST APPROXIMATION.

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1. **Introduction.** For many purposes, arbitrary rational functions are more useful in approximating to given analytic functions of a complex variable than are polynomials. For instance it is shown by Runge in his classical paper on approximation by polynomials[1] that a function $f(z)$ analytic in a closed region of the z-plane bounded by a finite number of non-intersecting Jordan curves can be uniformly approximated in that region as closely as desired by a rational function of z.[2] Such approximation by a *polynomial* may not be possible. It is the purpose of the present paper to show that in the study of two other phases of approximation it may also be more advantageous to use general rational functions than polynomials, namely 1) degree of approximation, that is, asymptotic properties of the measure of approximation of the sequence of functions of best approximation, and 2) overconvergence, the phenomenon that a sequence of functions approximating a given function in a given region frequently converges to that given function (or its analytic extension) not merely in the given region but also in a larger region containing the given region in its interior. The term overconvergence has recently been used by Ostrowski in a somewhat different connection.

A rational function of the form

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[1] Acta mathematica vol. 6 (1885), pp. 229—244.

[2] For more detailed results, compare Walsh, Mathematische Annalen vol. 96 (1926), pp. 437—450 and Transactions of the American Mathematical Society vol. 31 (1929), pp. 477—502.