

ALMOST PERIODICITY AND GENERAL TRIGONOMETRIC SERIES.¹

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Preface.

The class of Bohr's [1], [2], [3]² almost periodic functions may be considered from two different points of view. On the one hand it is the class of continuous functions possessing a certain structural property which is a generalisation of pure periodicity, and on the other it is the class of limit functions of uniformly convergent sequences of finite trigonometric polynomials. The main part of Bohr's theory of a. p. functions of a real variable developed in his first two papers [1], [2] was devoted to the proof of the identity of these two classes.

Further development of the theory of almost periodic functions was directed to generalisations of the theory. Corresponding to the two different points of view of the class of a. p. functions the generalisations went in two different directions. On the one hand there were further structural generalisations of pure periodicity. The first generalisations were very important ones given by W. Stepanoff [1], who succeeded in removing the continuity restrictions, and characterised the generalised almost periodicity not by values of the functions at each point, but by mean values over intervals of fixed length. N. Wiener studied almost periodicity and gave a new proof of the Fundamental Theorem by means of representation of functions by Fourier integrals [1], [2] and independently of Stepanoff he arrived at one of his (Stepanoff's) generalisations.

¹ The investigations in this paper were completed in a collaboration between the authors several years ago. The final redaction of the paper belongs to A. S. Besicovitch. An account of the principles of this paper was given by H. Bohr at the Congress at Bologna 1928 and in his paper [4].

² The list of papers referred to is given at the end of this paper.