ON APPELL'S DECOMPOSITION OF A DOUBLY PERIODIC FUNCTION OF THE THIRD KIND.

By

MIGUEL A. BASOCO

of PASADENA, CALIF.

1. Introduction.

Hermite[1] has defined a function $\varphi(z)$ to be doubly periodic of the third kind if it is meromorphic and satisfies two periodicity relations of the form

$$(1) \qquad \begin{cases} \varphi(z + 2\,\omega) = e^{a z + b} \cdot \varphi(z), \\ \varphi(z + 2\,\omega') = e^{c z + d} \cdot \varphi(z), \end{cases}$$

where $a, b, c, d, \omega, \omega'$ are constants and ω'/ω is a complex number $\alpha + i\beta$, $\beta \neq 0$. It can be shown that the properties of $\varphi(z)$ are deducible from those of a suitably defined function $F(z)$ which is likewise meromorphic and satisfies the simpler periodicity relations

$$(2) \qquad \begin{cases} F(z + \pi) \; = F(z), \\ F(z + \pi\,\tau) = e^{-2 m i z} F(z), \; i = \sqrt{-1}, \; m \neq 0, \end{cases}$$

where τ is a complex number with non-vanishing imaginary part, and m an integer, positive or negative. It can be proved that m is the excess of the number of zeros over the number of poles of the function in a period cell.

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[1] Hermite, Comptes Rendus, 1861, 1862; Journal für die reine und angewandte Mathematik; Band 100; Œuvres, tome II, p. 109; tome IV. p. 223.