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Mutually stationary sequences of sets and the non-saturation of the non-stationary ideal on $P_\varkappa(\lambda)$

by

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1. Introduction and preliminary remarks

An important combinatorial property of an ideal I on a set Z is the *saturation* of the ideal, i.e. the least upper bound on the cardinality of a well-ordered chain in the completion of the Boolean algebra $P(Z)/I$. This is particularly interesting in the case where I is a naturally defined ideal, such as the non-stationary ideal.

It is known to be consistent for the non-stationary ideal on ω_1 to be ω_2-saturated. This was shown first from strong determinacy hypotheses by Steel and Van Wesep [SV], and later from large cardinals by Foreman, Magidor and Shelah [FMS]. Shelah has the optimal result, showing this property consistent relative to the existence of a Woodin cardinal.

Shelah has shown that the non-stationary ideal on a successor cardinal $\varkappa>\omega_1$ can never be saturated, by showing that any saturated ideal on a successor cardinal must concentrate on a critical cofinality. Further, Gitik and Shelah (extending earlier work of Shelah) have shown that for $\varkappa$ a successor of a singular cardinal, the non-stationary ideal cannot be $\varkappa^+$-saturated even when restricted to the critical cofinality. They have also shown that the non-stationary ideal on an inaccessible cardinal can never be saturated.

Similar questions arise for the non-stationary ideal on $P_\varkappa(\lambda)$. Burke and Matsubara, using work of Cummings, Gitik and Shelah, were able to establish that the non-stationary ideal on $P_\varkappa(\lambda)$ is not λ^+-saturated in the cases where $\operatorname{cof}(\lambda)\neq\varkappa$, and when $\varkappa>\omega_1$ is a

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