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Thick points for planar Brownian motion and the Erdős–Taylor conjecture on random walk

by

AMIR DEMBO

Stanford University
Stanford, CA, U.S.A.

YUVAL PERES

University of California
Berkeley, CA, U.S.A.
and
Hebrew University
Jerusalem, Israel

JAY ROSEN

CUNY
Staten Island, NY, U.S.A.

and

OFER ZEITOUNI

Technion
Haifa, Israel

1. Introduction

Forty years ago, Erdős and Taylor [7] posed a problem about simple random walks in $\mathbf{Z}^2$: *How many times does the walk revisit the most frequently visited site in the first n steps?* Denote by $T_n(x)$ the number of visits of planar simple random walk to x by time n, and let $T_n^* := \max_{x\in\mathbf{Z}^2} T_n(x)$. Erdős and Taylor [7, (3.11)] proved that

$$\frac{1}{4\pi} \leqslant \liminf_{n\to\infty} \frac{T_n^*}{(\log n)^2} \leqslant \limsup_{n\to\infty} \frac{T_n^*}{(\log n)^2} \leqslant \frac{1}{\pi} \quad \text{a.s.,} \tag{1.1}$$

and conjectured that the limit exists and equals $1/\pi$ a.s. The importance of determining the value of this limit is clarified in (1.3) below, where this value appears in the power laws governing the local time of the walk.

The Erdős–Taylor conjecture was quoted in the book by Révész [19, §19.2], but to the best of our knowledge, the bounds in (1.1) were not improved prior to the present paper. As it turns out, an important step towards our solution of the Erdős–Taylor conjecture was the formulation by Perkins and Taylor [17] of an analogous problem on the maximal occupation measure that planar Brownian motion (run for unit time) can

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