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Quasiregular mappings and cohomology

by

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1. Introduction

The main result of this paper is the following statement.

THEOREM 1.1. *Let N be a closed, connected and oriented Riemannian n-manifold, $n\geqslant 2$. If there exists a nonconstant K-quasiregular mapping $f:\mathbf{R}^n\to N$, then*

$$\dim H^*(N)\leqslant C(n,K), \tag{1.2}$$

where $\dim H^*(N)$ is the dimension of the de Rham cohomology ring $H^*(N)$ of N, and $C(n,K)$ is a constant only depending on n and K.

As will be discussed shortly, Theorem 1.1 provides first examples of compact manifolds with small fundamental group that do not receive nonconstant quasiregular mappings from Euclidean space.

Recall that a continuous mapping $f:X\to Y$ between connected and oriented Riemannian n-manifolds, $n\geqslant 2$, is *K-quasiregular*, $K\geqslant 1$, if the first distributional derivatives of f in local charts are locally n-integrable and if the (formal) differential $Df(x)$: $T_xX\to T_{f(x)}Y$ satisfies

$$|Df(x)|^n\leqslant K\det Df(x) \tag{1.3}$$

for almost every $x\in X$. In (1.3), and throughout this paper, $|Df(x)|$ denotes the operator norm of the linear map $Df(x)$, and $\det Df(x)$ its determinant. We say that a mapping is *quasiregular* if it is K-quasiregular for some $K\geqslant 1$. The synonymous term a *mapping of bounded distortion* is also used in the literature.

Nonconstant quasiregular mappings are discrete (the preimage of each point is a discrete set) and open according to a deep theorem of Reshetnyak [Re1]. Thus, quasiregular mappings are generalized branched coverings with geometric control given by (1.3).

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