

# Spectral theory of Laplacians for Hecke groups with primitive character

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## Introduction

It was proved by [Sel] that the Laplacian  $A(\Gamma)$  for congruence subgroups  $\Gamma$  of the modular group  $\Gamma_{\mathbf{Z}}$  has an infinite sequence of embedded eigenvalues  $\{\lambda_i\}$  satisfying a Weyl law  $\#\{\lambda_i \leq \lambda\} \sim (|F|/4\pi)\lambda$  for  $\lambda \rightarrow \infty$ . Here  $|F|$  is the area of the fundamental domain  $F$  of the group  $\Gamma$ , and the eigenvalues  $\lambda_i$  are counted according to multiplicity. The same holds true for the Laplacian  $A(\Gamma; \chi)$ , where  $\chi$  is a character on  $\Gamma$  and  $A(\Gamma; \chi)$  is associated with a congruence subgroup  $\Gamma_1$  of  $\Gamma$ . It is an important question whether this is a characteristic of congruence groups or it may hold also for some non-congruence subgroups of  $\Gamma_{\mathbf{Z}}$ . To investigate this problem Phillips and Sarnak studied perturbation theory for Laplacians  $A(\Gamma)$  with regular perturbations derived from modular forms of

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