A NOTE ON DERIVATES AND DIFFERENTIAL COEFFICIENTS.

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§ 1.

The main theorem obtained in the present note is the following: — *Except at a countable set of points, the lower derivate on either side is not greater than the upper derivate on the other side;* i. e. using an accepted notation which explains itself[1]

$$f_-(x) \leq f^+(x),$$

and also

$$f_+(x) \leq f^-(x).$$

The primitive fonction $f(x)$ may be any function whatever of the single real variable x. If $f(x)$ is a continuous function, this theorem enables us to assert that, except at a countable set of points, $f(x)$ has at least one symmetric derivate, that is to say there is at least one sequence of positive, and one sequence of negative values of h, both with zero as limit, corresponding to each point x, such that the incrementary ration $(f(x+h)-f(x))/h$ has the same limit for the two sequences. I define accordingly *the mean symmetric derivate of a continuous function* $f(x)$ to be the trigonometric mean (§ 7) between the greatest and least symmetric derivate at each point; *the mean symmetric derivate of a continuous function then exists except at most at a countable set of points; it agrees with the differential coefficient, wherever this exists, and is finite except at a set of points of content zero.*

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[1] W. H. Young and the present author, »On Derivates and the Theorem of the Mean», 1908, Quart. Jour. of Pure and Applied Math., § 2, p. 4. Scheeffer, who first introduced the concept of a derivate, used $D_-f(x)$, etc. »Allgemeine Untersuchungen über Rectification der Curven», 1884, Acta Math. 5, p. 52.