On the parabolic kernel of the Schrödinger operator

by

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§ 0. Introduction

In this paper, we will study parabolic equations of the type

$$\left(\Delta - q(x,t) - \frac{\partial}{\partial t}\right) u(x,t) = 0 \tag{0.1}$$

on a general Riemannian manifold. The function $q(x, t)$ is assumed to be C^2 in the first variable and C^1 in the second variable. In classical situations [20], a Harnack inequality for positive solutions was established locally. However, the geometric dependency of the estimates is complicated and sometimes unclear. Our goal is to prove a Harnack inequality for positive solutions of (0.1) (§ 2) by utilizing a gradient estimate derived in § 1. The method of proof is originated in [26] and [8], where they have studied the elliptic case, i.e. the solution is time independent. In some situations (Theorems 2.2 and

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