Rigidity of time changes for horocycle flows

by

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Let T_t be a measure preserving (m.p.) flow on a probability space (X,μ) and let τ be a positive integrable function on X, $\int_X \tau\, d\mu=\bar{\tau}$. We say that a flow T_t^τ is obtained from T_t by the time change τ if

$$T_t^\tau(x)=T_{w(x,t)}(x)$$

for μ-almost every (a.e.) $x\in X$ and all $t\in\mathbf{R}$, where $w(x,t)$ is defined by

$$\int_0^{w(x,t)} \tau(T_u x)\,du=t.$$

The flow T_t^τ preserves the probability measure μ_τ on X defined by

$$d\mu_\tau(x)=(\tau/\bar{\tau})\,d\mu(x),\quad x\in X.$$

We say that two integrable functions $\tau_1,\tau_2\colon (X,\mu)\to\mathbf{R}$ are homologous along T_t if there is a measurable $v\colon X\to\mathbf{R}$ such that

$$\int_0^t (\tau_1-\tau_2)(T_u x)\,du=v(T_t x)-v(x)$$

for μ-a.e. $x\in X$ and all $t\in\mathbf{R}$. One can check that two time changes τ_1 and τ_2 are homologous via v if and only if (iff) the map $\psi_v\colon X\to X$ defined by

$$\psi_v(x)=T_{\sigma(x)}x,$$

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