

# ANALYTIC THEORY OF SINGULAR DIFFERENCE EQUATIONS.

By

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## § 1. Introduction.

In the present paper we study a linear difference system of order  $n$  in  $y_1(x), \dots, y_n(x)$ ,

$$(1) \quad y_i(x+1) = \sum_{j=1}^n a_{ij}(x) y_j(x)$$
$$(|a_{ij}(x)| \neq 0; i = 1, 2, \dots, n),$$

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