HARMONIC ESTIMATION IN CERTAIN SLIT REGIONS AND A THEOREM OF BEURLING AND MALLIAVIN

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Introduction

Suppose we form a domain $\mathcal{D}$ consisting of the union of the upper and lower half planes and a finite number of bounded open intervals from the real axis. The remainder, Γ, of the real axis is the boundary of $\mathcal{D}$. If $u(z)$ is subharmonic in $\mathcal{D}$, behaves reasonably well near Γ, and is dominated by $a|\text{Im } z|$ for $|z|$ large, and if, on Γ, the boundary values $u(x \pm i0)$ are known to be less than some given majorant $M(x)$, the possible size of u at any point of $\mathcal{D}$ is governed by two factors:

(i) The allowed rate of growth (or required rate of decrease, as the case may be) of $u(z)$ at ∞.

(ii) The magnitude of the majorant $M(x)$.

The interplay between these two factors is studied in Part I of the present paper. It is remarkable that in many cases their effects are comparable, and depend on the domain $\mathcal{D}$ solely through a quantity, called here the Selberg number, having a quite simple function-theoretic definition.

The results obtained in Part I are specific enough to yield a fairly straightforward duality proof of a theorem of Beurling and Malliavin [2] on the existence of certain kinds of multipliers for entire functions of exponential type. This application is given in Part II.

Part III contains an elementary derivation of another multiplier theorem of Beurling and Malliavin from the one proved in Part II. It can be read independently of the rest of the paper.

Work on the material presented here took several years, and was completed at the end of 1977 while I was staying at the Mittag-Leffler Institute in Sweden. I am very grateful to Professor L. Carleson of that institute for having let me discuss the ideas of