QUADRATIC DIFFERENTIALS AND FOLIATIONS

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Introduction

This paper concerns the interplay between the complex structure of a Riemann surface and the essentially Euclidean geometry induced by a quadratic differential.

One aspect of this geometry is the "trajectory structure" of a quadratic differential which has long played a central role in Teichmüller theory starting with Teichmüller's proof of the existence and uniqueness of extremal maps. Ahlfors and Bers later gave proofs of that result. In other contexts, Jenkins and Strebel have studied quadratic differentials with closed trajectories.

Starting from the dynamical problem of studying diffeomorphisms on a C^∞ surface M, Thurston [17] invented *measured foliations*. These are foliations with certain kinds of singularities and an invariantly defined transverse measure. A precise definition is given in Chapter I, § 1. This notion turns out to be the correct abstraction of the trajectory structure and metric induced by a quadratic differential. In this language our main statement says that *given any measured foliation F on M and any complex structure X on M, there is a unique quadratic differential on the Riemann surface X whose horizontal trajectory structure realizes F.* In particular any trajectory structure on one Riemann surface occurs uniquely on every Riemann surface of that genus.

In the special case when the foliation has closed leaves, an analogous theorem was proved by Strebel [15]. Earlier Jenkins [13] had proved that quadratic differentials with closed trajectories existed as solutions of certain extremal problems. We deduce Strebel's theorem from ours in Chapter I, § 3.

By identifying the space of measured foliations with the quadratic forms on a fixed Riemann surface, we are able to give an analytic and entirely different proof of a result of Thurston's [17]; that the space of projective classes of measured foliations is homeomorphic to a sphere. This is also done in Chapter I, § 3.

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