# ON THE CLASSIFICATION OF HOMEOMORPHISMS OF 2-MANIFOLDS AND THE CLASSIFICATION OF 3-MANIFOLDS

BY

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## Introduction

Given two topological spaces, is it possible to determine whether they are homeomorphic? This is the homeomorphism problem and most work in topology is directed toward some aspect of the homeomorphism problem. A plan for solving the homeomorphism problem for "most" 3-manifolds has been developed by Wolfgang Haken. However, a certain very special step in this plan has eluded proof. The problem of providing a proof for this special case amounts to the problem of classifying homeomorphisms of compact, orientable 2-manifolds. In this paper a method for classifying homeomorphisms of compact, orientable 2-manifolds will be given, and hence it will be possible to classify all compact, orientable, irreducible, boundary irreducible, sufficiently large 3-manifolds. Hence "most" 3-manifolds of interest can be classified, including all knot and link spaces.

Haken developed the theory in his series of papers: [1]–[5]. In [11], Schubert has explained the essential points of Haken's theory of normal surfaces. Waldhausen [12] has written a summary of the classification procedure, using the recent results of Johannson [6], [7].

### The conjugacy problem for self-homeomorphisms of compact, orientable surfaces

Let the surface $S$ be compact and orientable, and let $f$, $g$ be two homeomorphisms of $S$ onto itself. Assume that there exists a homeomorphism $h$ of $S$ onto itself such that $h^{-1}fh$ is isotopic to $g$. In this case, $f$ and $g$ are said to be conjugate. Given two homeomorphisms such as $f$ and $g$, the conjugacy problem asks whether or not they are conjugate. In order to complete Haken's program for the classification of sufficiently large 3-manifolds, we need to prove a result which is slightly stronger than the conjugacy problem. Namely, if $\partial S \neq \emptyset$ and if $f$ and $g$ agree on $\partial S$, then our problem is to determine whether $f$ and $g$