AN ELEMENTARY METHOD IN THE STUDY OF NONNEGATIVE CURVATURE

BY

H. WU [1]

University of California
Berkeley, Ca., USA

A standard technique in classical analysis for the study of continous sub-solutions of the Dirichlet problem for second order operators may be illustrated as follows. Suppose it is to be shown that a continuous real function $f(x)$ is convex (respectively, stricly convex) at x_0; then it suffices to produce a C^2 function $g(x)$ such that $g(x) \leqslant f(x)$ near x_0 and $g(x_0) = f(x_0)$, and such that $g''(x_0) \geqslant 0$ (respectively $g''(x_0) \geqslant$ some fixed positive constant). The main point of this procedure is to sidestep arguments involving continuous functions by working with differentiable functions alone. Now in global differential geometry, the functions that naturally arise are often continuous but not differentiable. Since much of geometric analysis reduces to second order elliptic problems, this technique then recommends itself as a natural tool for overcoming this difficulty with the lack of differentiability. In a limited way, this technique has indeed appeared in several papers in complex geometry (e.g. Ahlfors [1], Takeuchi [20], Elencwajg [7] and Greene–Wu [11]; cf. also Suzuki [19]). The main purpose of this paper is to broaden and deepen the scope of this method by making it the central point of a general study of nonnegative sectional, Ricci or bisectional curvature. The following are the principal theorems; the relevant definitions can be found in Section 1.

Let M be a noncompact complete Riemannian manifold and let $0 \in M$ be fixed. Let $\{C_t\}_{t \in I}$ be a family of closed subsets of M indexed by a subset I of $\mathbf{R}$. Assume that $e_t \equiv d(0, C_t) \to \infty$ as $t \to \infty$, where $d(p, q)$ will always denote the distance between $p, q \in M$ relative to the Riemannian metric. The family of functions $\eta_t \colon M \to \mathbf{R}$ defined by $\eta_t(p) =$

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