

MICRO-HYPERBOLIC SYSTEMS

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Table of Contents

Introduction	1
§ 1. Preliminaries	4
§ 2. Statement of the main theorems	8
§ 3. Action of microdifferential operators in the complex domain	11
§ 4. Prolongation theorem in the complex domain	20
§ 5. Micro-hyperbolic systems on the boundary of an open subset of a complex manifold	27
§ 6. Division theorems for sheaves of microfunctions with holomorphic parameters	34
§ 7. Proofs of the main theorems	39
§ 8. Application 1: Cauchy problem for sheaves of coherent \mathcal{E}_X -modules	42
§ 9. Application 2: Propagation of singularities	46
§ 10. Application 3: Holonomic systems	49
Bibliography	54

Introduction

The so-called “Cauchy Problem” has a very long and classical story, from J. Hadamard [7], I. G. Petrovski [22], J. Leray [20], L. Gårding [6], . . . to, for example, the last results of Ivrii and Pietkov [13] or L. Hörmander [12], but we will not review here.

The difficulty treated by the last authors lies in the fact that the principal symbol of the operator is not of simple characteristic, and the lower order symbols have an essential role (Levi conditions). However, we know by [4] that with the use of hyperfunctions the situation is simple: “hyperbolicity” is given by the principal symbol. This fact allows us to treat the case of (overdetermined) systems. In this paper, we treat the initial value problem and the problem of propagation for hyperfunction or microfunction solutions of (micro-)hyperbolic systems. The hyperbolicity is just a geometrical property of the complex characteristic variety of the system.