ON THE GREATEST PRIME FACTOR OF A QUADRATIC POLYNOMIAL

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1. Introduction

The theorem that, if P_x be the greatest prime factor of

$$\prod_{n \leqslant x} (n^2+1),$$

then

$$\frac{P_x}{x} \to \infty$$

as $x \to \infty$, for which as for many other interesting results in the theory of numbers we are indebted to Chebyshev, has attracted the interest of several mathematicians. Revealed posthumously as little more than a fragment in one of Chebyshev's manuscripts, the theorem was first published and fully proved in a memoir by Markov in 1895 [6], while later in the same year a generalisation by Ivanov [4] appeared in which the polynomial n^2+1 was replaced by n^2+A for any positive A (an account of both Markov's and Ivanov's work is to be found in Paragraphs 147 and 149 of Landau's *Primzahlen* [5]). In 1921 Nagell [7] improved and further generalised Chebyshev's theorem by shewing that for any $\varepsilon<1$ and for all sufficiently large x

$$\frac{P_x}{x} > \log^{\varepsilon} x,$$

where P_x is the largest prime factor in the product obtained by replacing n^2+1 by any irreducible integral non-linear polynomial $f(n)$. The final result is due to Erdös [1], who in 1952 improved Nagell's result by shewing that

$$\frac{P_x}{x} > (\log x)^{c_1 \log \log \log x}$$

by a method which he stated could be developed further to yield