ON THE SPECTRAL SYNTHESIS OF BOUNDED FUNCTIONS.

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1. Introduction.

In this paper L^1, L^2 and L^∞ will denote the linear metric spaces formed by the measurable functions over $-\infty < x < \infty$ which are respectively summable, of summable square or equivalent to a bounded function. The corresponding norms will be denoted by $\|\varphi\|_1$, $\|\varphi\|_2$ and $\|\varphi\|_\infty$.

To each $\varphi(x) \in L^\infty$ corresponds a closed set Λ_φ of real numbers, termed the spectral set of φ, which is formed, briefly, by those λ for which the pure oscillation $e^{i\lambda x}$ is contained in the manifold spanned by the set

$$\varphi(x+\tau) \qquad (-\infty < \tau < \infty) \tag{1.1}$$

in the weak topology of bounded functions, i. e. for every $G(x) \in L^1$ the condition

$$\int_{-\infty}^{\infty} \varphi(x+\tau)\, G(x)\, dx = 0 \qquad (-\infty < \tau < \infty)$$

implies [1]

$$\int_{-\infty}^{\infty} e^{i\lambda x}\, G(x)\, dx = 0.$$

The main problem of spectral Synthesis of L^∞ is to decide whether each $\varphi(x) \in L^\infty$ is contained in the weak closure of the manifold spanned by the oscillations

$$e^{i\lambda x} \qquad (\lambda \in \Lambda_\varphi). \tag{1.2}$$

[1] As is easily proved this definition leads to the same Λ_φ as that obtained by the stronger topology used by the author in a previous paper; *Un Théorème sur les fonctions bornées . . .*, Acta math. vol. 77, 1945.