

ON INJECTIVE BANACH SPACES AND THE SPACES $L^\infty(\mu)$ FOR FINITE MEASURES μ

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Introduction

We are interested here in the linear topological properties of those Banach spaces associated with injective Banach spaces. We study in particular detail, the spaces $L^\infty(\mu)$ for finite measures μ , and obtain applications of this study to problems concerning injective Banach spaces in general.⁽²⁾ (Throughout the rest of this introduction, “ μ ” and “ ν ” denote arbitrary finite measures on possibly different unspecified measurable spaces).

For example, we classify the spaces $L^\infty(\mu)$ themselves up to isomorphism (linear homeomorphism) in § 3, and all their conjugate spaces $((L^\infty(\mu))^*, (L^\infty(\mu))^{**}, (L^\infty(\mu))^{***}, \text{etc.})$

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⁽²⁾ It is easily seen that if λ is a σ -finite measure, then there exists a finite measure μ with $L^p(\lambda)$ isometric to $L^p(\mu)$ for all p , $1 \leq p < \infty$. Thus all of our results concerning finite measures generalize immediately to σ -finite measures.