INEQUALITIES FOR STRONGLY SINGULAR CONVOLUTION OPERATORS

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I. Introduction

Suppose that f is an L^p function on the torus $T^n = S^1 \times S^1 \times \ldots \times S^1$. Must the partial sums of the multiple Fourier series of f converge to f in the L^p norm? For the one-dimensional case, $T = S^1$, an affirmative answer has been known for many years. More specifically, suppose that $f \in L^p(S^1)$ has the Fourier expansion $f \sim \sum_{k=-\infty}^{\infty} a_k e^{ik\theta}$, and set $f_m(\theta) = \sum_{k=-m}^{m} a_k e^{ik\theta}$. Then f_m converges to f in $L^p(S^1)$, as $m \to \infty$ —provided $1 < p < +\infty$ (see [14]).

A whole slew of n-dimensional analogues of this theorem suggest themselves. Here are two natural conjectures.

(I) Let $f \in L^p(T^n)$ have the multiple Fourier expansion

$$f(\theta_1 \ldots \theta_n) = \sum_{k_1 \ldots k_n = -\infty}^{\infty} a_{k_1 \ldots k_n} e^{i(k_1\theta_1 + \ldots + k_n\theta_n)}.$$

For each positive integer m, set

$$f_m(\theta_1 \ldots \theta_n) = \sum_{|k_1| \leq m, |k_2| \leq m, \ldots, |k_n| \leq m} a_{k_1 \ldots k_n} e^{i(k_1\theta_1 + \ldots + k_n\theta_n)}.$$

Then $f_m \to f$ in $L^p(T^n)$, as $m \to \infty$.

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