# VALIRON DEFICIENT VALUES FOR MEROMORPHIC FUNCTIONS IN THE PLANE

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## 1. Introduction

**1.1.** *The function-theoretic problem.* This paper is concerned with a problem in the Nevanlinna theory of functions meromorphic in $|z| < \infty$ (referred to in the sequel simply as functions). We shall assume acquaintance with the standard notation of the Nevanlinna theory (cf. [3], [5] or [10])

$$T(r, f), \quad N(r, a), \quad m(r, a)$$

and with Nevanlinna's fundamental theorems (see [5] pp. 5, 31 or [10] pp. 168, 243). The *Valiron deficiency* $\Delta(a, f)$ of a value $a$ for the function $f = f(z)$ is, by definition,

$$\Delta(a, f) = \limsup_{r\to\infty} \frac{m(r, a)}{T(r, f)} = \limsup_{r\to\infty} \frac{T(r, f) - N(r, a)}{T(r, f)}.$$

If $\Delta(a, f) > 0$ for a particular value of $a$, then that value is said to be *Valiron deficient* for the function $f$.

We here investigate the size of the set of Valiron deficient values, when the function $f$ is given.

To say that a function value $a$ is Valiron deficient means, roughly, that it is assumed significantly less often in $|z| \leq r$ than are other function values, for some sequence $r = r_n \to \infty$. If this is the case for all sufficiently large $r$-values, then $a$ is Nevanlinna deficient.

1.2. *Some known results.* A meromorphic function can have at most countably many values $a$ which are Nevanlinna deficient ($\liminf m(r, a)/T(r, f) > 0$), but Valiron has constructed an example of an entire function of order one such that the set of (what is now called) Valiron deficient values has "effectivement la puissance du continu" and hence is non-countable (cf. [12] pp. 263–266).