

A CONTRIBUTION TO THE THEORY OF DIVERGENT SEQUENCES.¹

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In this paper we define and examine a new method of summation which assigns a general limit $\text{Lim } x_n$ to certain bounded sequences $x = \{x_n\}$. This method is analogous to the mean values which are used in the theory of almost periodic functions, furthermore it is narrowly connected with the limits of S. BANACH.² The sequences which are summable by this method F we shall call almost convergent. In spite of the fact that our method contains certain classes of matrix methods (for bounded sequences) it is not strong (§ 3). Its most remarkable property is that most of the commonly used matrix methods contain the method F (§ 5). In spite of this F is equivalent to none of the matrix methods (§ 7). In § 6 we shall examine a certain class of matrix methods and compare them with the method F .¹

§ 1. Different Definitions of the Method F .

Let M be the entity of all bounded sequences of real numbers $x = \{x_n\}$. M is a Banach space, if we there define the linear operations in a natural manner and the norm of an element $x = \{x_n\}$ by

$$\|x\| = \sup_n |x_n|.$$

Then evidently the set C of all convergent sequences is a linear subspace of M . S. BANACH proved the existence of certain functions of the element $x = \{x_n\}$ in

¹ Some preliminary results have been published in Zapiski Univ. Leningrad, Math. Ser. 12, 30—41 (1941).

² Cf. BANACH, Théorie des opérations linéaires, Warszawa 1932, p. 33—34.