NON-HOMOGENEOUS TERNARY QUADRATIC FORMS.

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1. This work has arisen from the consideration of possible extensions of Minkowski's theorem on the product of two non-homogeneous linear forms. If

$$L_1 = \alpha x + \beta y, \qquad L_2 = \gamma x + \delta y$$

are two linear forms with real coefficients, and c_1, c_2 are any two real numbers, Minkowski's theorem asserts that there exist integers x, y such that

$$(1) \qquad |(L_1 + c_1)(L_2 + c_2)| \leqq \tfrac{1}{4}\Delta,$$

where $\Delta = |\alpha\delta - \beta\gamma|$, and we suppose $\Delta \neq 0$. It is conjectured that a similar result holds for the product of n non-homogeneous linear forms in n variables, with 2^{-n} in place of $\frac{1}{4}$. So far this conjecture has been proved only for $n = 3$, by Remak, and for $n = 4$, by Dyson.

Minkowski's theorem can be stated in another form, which suggests other possible extensions. Write

$$L_1 L_2 = a x^2 + b x y + c y^2 = Q(x, y);$$

then $Q(x, y)$ is an indefinite binary quadratic form with discriminant

$$b^2 - 4ac = \Delta^2.$$

Determine real numbers x_0, y_0 so that

$$c_1 = \alpha x_0 + \beta y_0, \qquad c_2 = \gamma x_0 + \delta y_0.$$

Then Minkowski's theorem asserts that for any indefinite binary quadratic form $Q(x, y)$, and any real x_0, y_0, there exist integers x, y such that

$$(2) \qquad |Q(x + x_0,\ y + y_0)| \leqq \tfrac{1}{4}\Delta.$$