

ANALYTIC THEORY OF LINEAR DIFFERENTIAL EQUATIONS.

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Index.

- 1°. Introduction.
1. Preliminary Facts.
2. The Q , Q_x and C_x Curves and Regions R .
3. A Lemma concerning Regions R_i .
4. Formal Integration.
5. Analytic Integration.
6. Iterations.
7. The Fundamental Existence Theorem.
8. Extension of the Regions of Validity of the Asymptotic Relations.
9. Converse Problems.

1°. **Introduction.** Our present object is to develop, *on the basis of the formal solutions and without any restrictions on the roots of the corresponding characteristic equation*, the analytic theory of a linear differential equation of order n

$$(A) \quad L_n(y) \equiv a_0(x)y^{(n)}(x) + a_1(x)y^{(n-1)}(x) + \cdots + a_{n-1}(x)y^{(1)}(x) + a_n(x)y(x) = 0$$
$$[a_0(x) \not\equiv 0; \quad a_n(x) \not\equiv 0]$$

from the point of view of the asymptotic nature of the solutions. Such a study will be given for the neighborhood of a singular point (regular or irregular). This point will be taken at infinity. The coefficients in (A) will be supposed to be analytic for $|x| \geq \varrho$ ($|x| \neq \infty$), being representable by convergent series of the form

$$(1) \quad a(x) = a_M x^{\frac{M}{p}} + a_{M-1} x^{\frac{M-1}{p}} + \cdots + a_1 x^{\frac{1}{p}} + a_0 + a_{-1} x^{-\frac{1}{p}} + a_{-2} x^{-\frac{2}{p}} + \cdots,$$