

A parametrized index theorem for the algebraic K -theory Euler class

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Contents

0. Introduction	1
Part I. An index theorem	8
1. Characteristics	8
2. Excisive characteristics and characteristic classes	10
3. The index theorem	18
4. Disk bundles and the unit transformation	28
5. The transfer of Becker–Gottlieb and Dold	36
Part II. Applications	39
6. Homotopy invariant characteristics	40
7. An excisive characteristic	46
8. Riemann–Roch theorems	54
Part III. Converse Riemann–Roch theory	66
9. Waldhausen’s theorems in h -cobordism theory	66
10. Converse Riemann–Roch for topological manifolds	79
11. Stability matters	90
12. Converse Riemann–Roch for smooth manifolds	92
References	100

0. Introduction

The classical setting. Suppose that $p: E \rightarrow B$ is a bundle of smooth compact manifolds. By this we mean that p comes with an atlas of local trivializations $\varphi_i: p^{-1}(U_i) \rightarrow U_i \times M_i$, where each M_i is a smooth compact manifold, the changes of charts

$$\varphi_j \varphi_i^{-1}: (U_i \cap U_j) \times M_i \rightarrow (U_i \cap U_j) \times M_j$$

are fiberwise smooth and the induced maps $(U_i \cap U_j) \times T^r M_i \rightarrow (U_i \cap U_j) \times T^r M_j$ are continuous for each $r > 0$.