THE HYPERCENTER OF A GROUP.

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The hypercenter of a finite group may be characterized by various properties which, however, cease to be equivalent if applied to infinite groups. Of the possibilities thus arising we investigate here only one, the terminal member of the upper central chain; and our problem is the intrinsic characterization of the normal subgroups contained in this hypercenter. These hypercentral subgroups may be defined as exactly those subgroups N of G which satisfy the following simple condition: If M is a normal subgroup of G and if $M < N$, then N/M contains a center element, not 1, of G/M.

Of the fundamental properties of hypercentral subgroups N of G the following seem to be outstanding: (a) if the normal subgroup M of G is a proper part of N, if x is an element of order a power of p in N/M and if g is some element in G/M, then there exists an integer m such that $xg^{p^m} = g^{p^m}x$; (b) if T is a subgroup of G such that $T < NT$, then the normalizer of T in NT is different from T; (c) if the normal subgroup M of G is a proper part of N, then there exists a normal subgroup V of G such that $M < V \leq N$ and such that M is the intersection of all the normal subgroups X of G which satisfy: $M \leq X < V$ and V/X is a finite minimal normal subgroup of G/X. Actually it will be shown that each of the two combinations (a, c) and (b, c) is characteristic for hypercentrality.

One of the most interesting phenomena encountered in the course of this investigation is the fact that hypercentral subgroups are never "very infinite". To make this rather vague statement more precise we mention two results: The maximum condition is satisfied by the subgroups of every finitely generated subgroup of the hypercenter; and finitely many elements of finite order in the hypercenter generate a finite subgroup. The latter remark points to a fascinating undercurrent of complications arising from encounters with Burnside's celebrated conjecture which had to be either circumvented or, in rather special instances, proved. The preparatory discussions of section 1 are very much concerned with just this situation; and some concepts and results of independent interest may be found there.

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