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A variational method in image segmentation: Existence and approximation results

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Introduction

The main input in computer vision is the image of a scene, given by the grey level of each point of the screen. This determines a real valued measurable function g on a plane domain Ω, which, in general, is discontinuous along the lines corresponding to the edges of the objects. Other discontinuities of g can be caused by shadows, surface markings, and possible irregularities in the surface orientation of the objects.

For all these reasons, when one wants to regularize g in such a way to eliminate the details of the scene which are too small and meaningless, one can expect to obtain a better approximation by means of a piecewise smooth function rather than by a globally smooth function.

This motivates the so called "segmentation problem", which is one of the main problems in image analysis: find a closed set K, made up of a finite number of regular arcs, and a smooth function u on $\Omega\setminus K$, such that

(S1) u varies smoothly on each connected component of $\Omega\setminus K$,

(S2) u is a good approximation of g on $\Omega\setminus K$.

The set K will be the union of the lines which give the best essential description of the image. The parameters which make such a description more or less good are the way in which (S1) and (S2) are satisfied and the minimality of K, expressed by the further requirement that

(S3) the total length of K is sufficiently small.

For a general treatment of this subject we refer to A. Rosenfeld and A. C. Kak [24]. Many problems in image segmentation can be solved by minimizing a functional depending on K and u, as pointed out by S. and D. Geman [15] for a similar problem defined on a lattice instead of a plane domain. The role of the functional to be minimized is to measure to what extent conditions (S1), (S2), and (S3) are satisfied.