L^∞ estimates for the $\bar\partial$ problem in a half-plane

by

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§ 1. Introduction

Suppose μ is a σ-finite complex-valued measure on the upper half-plane $\mathbf{R}^2_+=\{z=x+iy: y>0\}$. Then μ is called a Carleson measure if

$$\sup_I \frac{1}{|I|}|\mu|(I\times(0,|I|]) = \|\mu\|_C < \infty,$$

where the above supremum is taken over all intervals $I\subset\mathbf{R}$, and where $|\cdot|$ denotes one-dimensional Lebesgue measure. Invoking a fundamental theorem due to Carleson [6], Hörmander [21] showed that the $\bar\partial$ problem $\bar\partial F=\mu$ has a solution F satisfying

$$\|F\|_{L^\infty(\mathbf{R})} \leq C_0\|\mu\|_C$$

where μ is a Carleson measure. (Here and throughout the paper we denote by C_0 various universal constants.) The proof of this was based on the duality between H^1 and L^∞/H^∞ and the fact that

$$\|f^*\|_{L^p} \leq C_0\|f\|_{H^p}$$

where

$$f^*(t) = \sup_{|x-t|<y} |f(x+iy)|.$$

Here H^p, $0<p<\infty$, denotes the classical (holomorphic) Hardy space of functions holomorphic on $\mathbf{R}^2_+$ and satisfying

$$\sup_{y>0}\left(\int_{-\infty}^{\infty} |f(x+iy)|^p\,dx\right)^{1/p} = \|f\|_{H^p} < \infty.$$

(1) N.S.F. Grant MCS-8102631.