

# Fixed point free actions on $\mathbf{Z}$ -acyclic 2-complexes

by

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In this paper, we give a complete description of the finite groups which can act on 2-dimensional  $\mathbf{Z}$ -acyclic complexes without fixed points. One example of such an action (by the group  $A_5$ ) has been known for a long time, but as far as we know it is the only such action constructed earlier. In fact, we construct here actions of this type for many different finite simple groups.

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