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Analyticity of intersection exponents for planar Brownian motion

by

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1. Introduction

The goal of the present paper is to show that the intersection exponents for planar Brownian motions are analytic. Let $k \geqslant 1$ be a positive integer and let $X^1, ..., X^k$ be independent Brownian motions in the complex plane $\mathbf{C}$ started from 0. Let $Y, Y^1, Y^2, ...$ denote other independent planar Brownian motions started from 1, and let Ξ_t be the random variable (measurable with respect to $X^1, ..., X^k$)

$$\Xi_t = \mathbf{P}[Y[0,t] \cap (X^1[0,t] \cup ... \cup X^k[0,t]) = \varnothing \mid X^1[0,t] \cup ... \cup X^k[0,t]].$$

Note that

$$\mathbf{P}[(X^1[0,t] \cup ... \cup X^k[0,t]) \cap (Y^1[0,t] \cup ... \cup Y^p[0,t]) = \varnothing] = \mathbf{E}[\Xi_t^p].$$

The intersection exponent $\xi(k, \lambda)$ is defined for $\lambda > 0$ by

$$\mathbf{E}[\Xi_t^\lambda] \approx t^{-\xi(k,\lambda)/2}, \quad t \to \infty, \tag{1.1}$$

that is,

$$\xi(k, \lambda) := -2 \lim_{t \to \infty} \frac{\log \mathbf{E}[\Xi_t^\lambda]}{\log t}.$$

The existence of such exponents follows easily from a subadditivity argument. For a more detailed account of the definition and properties of these exponents, we refer the reader to our earlier papers [16], [12], [13]. Let us mention, however, that they are related to other critical exponents arising in statistical physics, including those predicted by theoretical physicists for planar critical percolation and self-avoiding walks (see references in [12]).

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