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Approximation of zonoids by zonotopes([1])

by

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1. Introduction

Consider the Euclidean ball B^n in $\mathbf{R}^n$. It is well known that B^n can be approximated in the Hausdorff metric by a sum of segments. Given $\varepsilon>0$, what is the number N needed so that the Hausdorff distance between B^n and a sum of segments $\Sigma_{j=1}^N I_j$ is less than ε? It is quite clear that $N=\exp(c(\varepsilon)n)$ for a suitable $c(\varepsilon)$ will suffice. The surprising fact is that actually $N=c(\varepsilon)n$ will do. This was proved in [F.L.M.]. In this paper we show that this fact is not a special property of B^n but that essentially the same holds for any convex body in $\mathbf{R}^n$ which is a limit of a sum of segments. Questions related to this topic have been studied in the literature till now mainly in the framework of Banach space theory. Also the main tools we use in this paper are taken from Banach space theory. In order to make the paper accessible to experts in convexity theory as well as those in Banach space theory we include in the paper somewhat more than the usual amount of background material.

The introduction is divided into two parts. We first explain the geometric problem and state the main results. We then pass on to the functional analytic formulation, survey the history of the problem and explain the contents of the various sections of this paper.

A zonotope in $\mathbf{R}^n$ is a polytope P which is a vector sum of segments $\{I_j\}_{j=1}^N$, i.e.,

$$P=\left\{x;x=\sum_{j=1}^N x_j,\ x_j\in I_j,\ j=1,\dots,N\right\}.$$

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