Acta Math., 183 (1999), 191–253

Asymptotic distribution of resonances for convex obstacles

by

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1. Introduction and statement of results

The purpose of this paper is to give asymptotics for the counting function of resonances for scattering by strictly convex $\mathcal{C}^{\infty}$-obstacles satisfying a pinched curvature condition. We show that the resonances lie in cubic bands and that they have Weyl asymptotics in each band. The asymptotics are in fact the same as those for eigenvalues of the Laplacian on the surface of the obstacle. The closer the pinching condition brings the obstacle to the ball the larger is the number of bands to which our result can be applied. Figure 1 illustrates the main theorem for the first band.

Heuristically, the resonances for convex bodies are created by waves creeping along the geodesics on the boundary and losing energy at a rate depending on the curvature. Consequently, the precise distribution depends in a subtle way on the dynamics of the geodesic flow of the surface and its relation to the curvature. A rigorous indication of that (for the case of analytic obstacles) was given by the first author in [25]. However, those subtle effects are mostly present in the distribution of imaginary parts of the resonances. The crude heuristic picture suggests that as far as the real parts are concerned the distribution should be governed by the same rules as those for eigenvalues of the surface, and our result justifies this claim. For our proof the pinching condition for the curvature needs to be imposed to eliminate interference between different bands.

The subject of locating and estimating resonances for convex bodies has a very long tradition. Its origins lie with the study of diffraction by spherical obstacles. The resonances of a ball are given by zeros of Hankel functions, and the study of the distribution of those zeros was conducted by Watson [36], Olver [19], Nussenzveig [18] and others. For more general convex obstacles they were then studied by Fock, Buslaev, Babich–Grigoreva [1], and more recently by Bardos–Lebeau–Rauch [2], Popov [20], Filippov–