A proof of the Bieberbach conjecture

by

LOUIS DE BRANGES

Purdue University
Lafayette, IN, U.S.A.

In 1916, L. Bieberbach [2] conjectured that the inequality

$$|a_n| \leqslant n|a_1|$$

holds for every power series $\Sigma_{n=1}^{\infty} a_n z^n$ with constant coefficient zero which represents a function with distinct values at distinct points of the unit disk. He also conjectured that equality holds with $n>1$ only for a constant multiple of the Koebe function

$$\frac{z}{(1+\omega z)^2}$$

where ω is a constant of absolute value one.

Bieberbach [2] verified the Bieberbach conjecture for the second coefficient. The Bieberbach conjecture for the third coefficient was verified by K. Löwner [9] in 1923. In 1955, P. R. Garabedian and M. Schiffer [7] verified the Bieberbach conjecture for the fourth coefficient. The Bieberbach conjecture for the sixth coefficient was verified in 1968 by R. N. Pederson [13] and, independently, by M. Ozawa [12]. In 1972, Pederson and Schiffer [14] verified the Bieberbach conjecture for the fifth coefficient. No other case of the Bieberbach conjecture has previously been verified.

A proof of the Bieberbach conjecture is now obtained for all remaining coefficients. Two other conjectures are also verified.

In 1936, M. S. Robertson [17] conjectured that the inequality

$$|\beta_1|^2+|\beta_2|^2+\ldots+|\beta_n|^2 \leqslant n|\beta_1|^2$$

holds for every odd power series $\Sigma_{n=1}^{\infty} \beta_n z^{2n-1}$ which represents a function with distinct values at distinct points of the unit disk. Such a power series is obtained from any

10–858285 *Acta Mathematica* 154. Imprimé le 27 Fevrier 1985