# ON CONVERGENCE AND GROWTH OF PARTIAL SUMS OF FOURIER SERIES

BY

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## 1. Introduction

In the present paper we shall introduce a new method to estimate partial sums of Fourier series. This will give quite precise results and will in particular enable us to solve the long open problem concerning convergence a.e. for functions in $L^2$. We denote by $s_n(x)$ the $n$th partial sum of a function $f(x) \in L^1(-\pi, \pi)$ and have the following theorem.

THEOREM. (a) *If for some* $\delta > 0$

$$\int_{-\pi}^{\pi} |f(x)| (\log^+ |f(x)|)^{1+\delta} dx < \infty, \tag{1.1}$$

*then*

$$s_n(x) = o(\log \log n), \quad a.e.$$

(b) *If* $f(x) \in L^p$, $1 < p < 2$, *then*

$$s_n(x) = o(\log \log \log n), \quad a.e.$$

(c) *If* $f(x) \in L^2$, *then* $s_n(x)$ *converges a.e.*

*Remarks.* (a) This result should be compared with Kolmogorov's example of an a.e. divergent series in $L^1$. If we consider in detail the construction of Hardy–Rogosinski (see [1], pp. 306–308), we see that the following is actually true. Given $\varepsilon(n) \to 0$, $n \to \infty$, there is a function $f \in L^1$ such that

$$s_n(x) \neq O(\varepsilon(n) \log \log n), \quad a.e.$$

The best previous result in this case is $o(\log n)$.

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